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Resit exam Robust Control

29 January 2026, 11:45–13:45

The exam consists of 4 exercises. Please write clearly and provide motivations for all your answers. You get 4 points for free and the maximum possible number of points is 40. Your grade is equal to the number of points divided by 4. Good luck!

1 (9 points) — Algebraic Riccati equation

Let $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{p\times n}$, with (A, B) stabilizable and (C, A) detectable. Consider the algebraic Riccati equation:

$$A^T P + PA - PBB^T P + C^T C = 0.$$

Let P be a real symmetric positive semidefinite solution. Prove that $\sigma(A - BB^T P) \subset \mathbb{C}^-$.

2 ($3 + 4 + 2 = 9$ points) — Computation of the H_2 performance

Consider the system Σ of the form

$$\dot{x}(t) = Ax(t) + Ed(t), \;\; z(t) = Cx(t).$$

We assume throughout this exercise that the matrix A is Hurwitz. The H_2-performance of Σ is given by $J(\Sigma) = \int_0^\infty ||T(t)||^2 dt$, where $T(t) = Ce^{At}E$ is the impulse response matrix and $||\cdot||$ denotes the Frobenius norm.

(a) Prove that there exists a symmetric positive semidefinite solution P to the Lyapunov equation

$$A^\top P + PA + C^\top C = 0. \tag{1}$$

Hint: consider the matrix $P = \int_0^\infty e^{A^\top t} C^\top C e^{At} dt$.

(b) Prove that the latter P is the *unique* symmetric solution to (1).

(c) Show that $J(\Sigma) = \operatorname{trace}(E^\top PE)$.

3 (4 + 5 = 9 points) — H_∞ control problem

Consider the system

$$\dot{x}(t) = Ax(t) + Bu(t) + Ed(t),\ y(t) = C_1x(t) + D_{12}d(t),\ z(t) = C_2x(t) + D_{21}u(t) + D_{22}d(t).$$

Here x is the state, u is the control input, d is the disturbance, y is the measured output and z is the performance output.

(a) Explain in detail what is meant by the H_∞ control problem associated with the given system and tolerance $\gamma > 0$.

(b) Explain how the bounded real lemma is used to obtain necessary and sufficient conditions for solvability of the H_∞ control problem. You do not need to write down all the formulas, just the main ideas.

4 (3 + 3 + 3 = 9 points) — Small gain theorem

For $i = 1, 2$ consider the systems Σ_i given by

$$\dot{x}_i(t) = A_ix_i(t) + B_iu_i(t),\ y_i(t) = C_ix_i(t) + D_iu_i(t),$$

where $u_i(t) \in \mathbb{R}^{m_i}$ and $y_i(t) \in \mathbb{R}^{p_i}$ and where it is assumed that $m_2 = p_1$ and $m_1 = p_2$.

(a) Explain what we mean by saying that the feedback interconnection $\Sigma_1 \times \Sigma_2$ of Σ_1 and Σ_2 is well-posed.

(b) Suppose that the interconnection is well-posed. Write down a state-space representation of $\Sigma_1 \times \Sigma_2$.

(c) Give a precise formulation of the small gain theorem for systems Σ_1 and Σ_2.

End of exam, cheers!